Quantum optical weak measurements can visualize photon dynamics in real time

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An experiment is proposed to visualize stroboscopically in real time the dynamics of a photon oscillating between two cavities. The visualization is implemented by a sequence of weak measurements (POVM), which are carried out by probing one of the cavities with a Rydberg atom and detecting a resulting phase shift by Ramsey interferometry. This way to measure the number of photons in a cavity was experimentally realized by Brune et al. . We suggest a feedback mechanism which minimizes the disturbance due to the measurement and enables a detection of the original evolution of the radiation field.

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There is much experimental progress in trapping single atoms, ions and photons. It has recently been reported that it is possible to trap individual atoms with a single photon in a cavity and to reconstruct the trajectory of the atom [1]. We want to contribute to this rapidly growing field of quantum visualization by proposing an experimental realization in a very "clean" setting. In a preceeding paper [2] we have shown theoretically and numerically that it is feasible to monitor in real time a dynamical process occurring in an individual two-level system with state vector

$$|\tilde{\psi}(t)\rangle = \tilde{c}_1(t)|\varphi_1\rangle + \tilde{c}_2(t)|\varphi_2\rangle \ . \tag{1}$$

Our aim in the following is to describe an experimental set up for quantum visualization which serves to register the time behavior of $|\tilde{c}_2(t)|^2$ approximately, while only weakly influencing the original dynamics of $|\tilde{\psi}(t)\rangle$. Here we assume the original dynamics to be known before the measurements. Why should we want to measure a theoretically known dynamics? Because we want to verify that our measurement procedure works. It is a non-trivial task to measure $|\tilde{c}_2(t)|^2$ – a quantity which refers to an ensemble – by a one run measurement on a single system in real-time. In order to achieve this goal we need only little a priori information about the original dynamics. In the concrete example of Rabi oscillations presented in this paper we succeed with the proposed measurement scheme knowing only the order of magnitude of the oscillation period. Our long-term objective is to devise a measurement scheme for the real time monitoring of partly unknown dynamics.

In classical physics it is possible to track the evolution of an individual system without disturbing it substantially. How can this aim at least approximately be achieved for quantum systems? Since projection measurements, which are also called sharp measurements, severely alter the original motion of $|\tilde{\psi}(t)\rangle$, they are not suitable in an one shot situation where only a single realization of this motion is available. An exception are QND schemes. They have the disadvantage to require for non-trivial dynamics an observable with a continuous spectrum [3], which does not exist for two-level systems. In case of a two-level system one needs instead weak (or unsharp) measurements (POVM measurements) by which the state of the system is less disturbed but nevertheless some information about the state is provided. A single weak measurement can be realized by suitably entangling the two-level system with a quantum meter via a unitary transformation (premeasurement) followed by a projection measurement on the meter. The latter supplies a measurement result, which is read off.

To track the development of $|\tilde{c}_2(t)|^2$ in time, a sequence of weak measurements is necessary. The corresponding series of measurement results can then be appropriately processed in real time to give the final measurement readout. Two conditions may thus be fulfilled simultaneously: i.) The back action of the measurements does only moderately disturb the original dynamics of the system given by the evolution of $|\tilde{\psi}(t)\rangle$ and ii.) the variance of the measurement results is small enough to enable a reliable estimate of the original time behavior of $|\tilde{c}_2(t)|^2$. Of course there is no information gain without disturbance of the state. In fact the greater the information gain the greater is the change of the state due to the measurement.

In this paper we sketch an experiment to visualize Rabi oscillations with frequency Ω_R. When fulfilling the conditions i.) and ii.) we have some freedom in adjusting the measuring apparatus. For example Ω_R does not have to be known exactly beforehand. An apparatus tuned to any frequency out of the interval $[0.75\,\Omega_R\,,\,1.5\,\Omega_R]$ would reveal Ω_R as the actual frequency (see below).

In cases in which a sequential measurement may be approximately treated as a continuous measurement it can in the selective regime be described by a stochastic master equation [4]. This powerful calculational tool enables an optimal exploitation of the information contained in the readout of the continuous measurement [5].

We are considering instead a series of well separated weak measurements. Such a series was employed to carry out a QND measurement of small photon numbers in an experiment of Brune, Haroche et al. [6], which was theoretically analyzed in [7] and experimentally realized in [8]. While the weakness of the measurements has been considered as an obstacle there, it turns out to be an advantage when it comes to the detection of dynamics. The experimental setup we sketch in the following is based on

the Brune-Haroche experiment. We have added a second cavity and a feedback mechanism. The system consists of one photon with frequency ω shared by two equally constructed, coupled cavities $\mathcal{C}_1$ and $\mathcal{C}_2$. One could think of two identical cavities connected by a waveguide (cf. [9]) or a transmissive mirror. For calculations the cavities are assumed to have infinite damping time. A justification will be given bellow. Their coupling is modeled by the interaction Hamiltonian of the Jaynes-Cummings type:

$$H = \hbar g (a_1 a_2^\dagger + a_1^\dagger a_2) \tag{2}$$

with coupling constant g. In the interaction picture, which we are going to use, (2) is the full Hamiltonian. The indices refer to the cavity numbers. Such a coupling of two cavities has also been considered in [10]. The photon which is delocalized over the two cavities can be described as a superposition of two states:

$$|\tilde{\psi}(t)\rangle = \tilde{c}_1(t)\,|1,0\rangle + \tilde{c}_2(t)\,|0,1\rangle\,. \tag{3}$$

The first and the second slot in the ket represent the number of photons in cavity $\mathcal{C}_1$ and cavity $\mathcal{C}_2$ respectively. For the initial state $|\tilde{\psi}(t=0)\rangle = |1,0\rangle$ we find Rabi-oscillations with the Rabi-frequency $\Omega_R := 2g$

$$|\tilde{c}_2(t)|^2 = \sin^2(gt)\,. \tag{4}$$

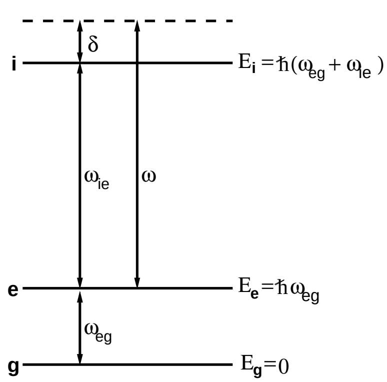

FIG. 1. Relevant levels of Rydberg atom

It is our goal to measure this original evolution of $|\tilde{c}_2(t)|^2$ in real time by probing the coupled cavities with atoms. We sent a Rydberg atom with three effective energy levels g, e, i (see Fig.1) and velocity v through the first cavity $\mathcal{C}_1$ (cp. [6,7]). The passage time L_c/v (L_c is the cavity length) is assumed to be much shorter than the period $T_R := 2\pi/\Omega_R = \pi/g$ of the oscillations of $|\tilde{c}_2(t)|^2$. Then the coupling of the two cavities is negligible during the time the atom spends in the cavity. The detuning of the atomic transitions with respect to the frequency of the cavity mode ω is such that the interaction between the atom and $\mathcal{C}_1$ is dispersive and only the energy levels e and i suffer an appreciable dynamical stark shift. Provided the atom enters the cavity in in a superposition of states $|g\rangle$ and $|e\rangle$, the effective Hamiltonian reads (cp. eqn. (16) in [7]):

$$H_{\text{int}} = \frac{\hbar\Omega^2}{\delta}\,|e\rangle\langle e| \otimes a_1^\dagger a_1\,, \tag{5}$$

where $\delta := \omega - \omega_{ie}$ and $\Omega = \overline{\Omega(r)}$ is the Rabi frequency averaged over the path of the atom through the cavity. With (5) the state of the enlarged system composed of the atom and the photon field changes according to

$$\begin{aligned}(c_e|e\rangle + c_g|g\rangle) \otimes |\psi\rangle \to c_e|e\rangle \otimes U_{\mathcal{C}_1}|\psi\rangle \\ + c_g|g\rangle \otimes |\psi\rangle\end{aligned} \tag{6}$$

with $U_{\mathcal{C}_1}$ being diagonal in the basis $|1,0\rangle$ and $|0,1\rangle$:

$$U_{\mathcal{C}_1} := e^{-i\varepsilon_1}|1,0\rangle\langle 1,0| + |0,1\rangle\langle 0,1|\,, \tag{7}$$

and $\varepsilon_1 := \frac{\Omega^2}{\delta}\frac{L_{\mathcal{C}}}{v}$. $|\psi\rangle$ represents the state of the 1-photon-field probed by atoms. The net effect of the atom-field coupling described by the interaction Hamiltonian (5) is that only the amplitude of the alternative $|e\rangle \otimes |1,0\rangle$ suffers a phase shift $e^{-i\varepsilon_1}$ while the amplitudes of the other quantum alternatives remain unchanged.

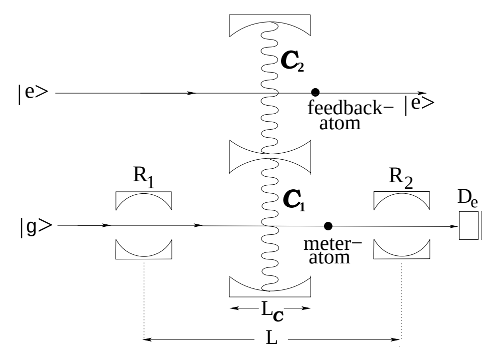

FIG. 2. Experimental setup

Phase shifts between several quantum alternatives may be measured by interferometry. As proposed in [6] it is convenient to use the Ramsey method of separated oscillatory fields (see Fig. 2). To this end a Rydberg atom is initially prepared in state $|g\rangle$. Before entering cavity $\mathcal{C}_1$ the state of the atom is transformed into a superposition of states $|e\rangle$ and $|g\rangle$ by entering a cavity which contains a first classical oscillatory microwave field R_1 with frequency ω_r. In the cavity $\mathcal{C}_1$ the atomic state becomes entangled with the state of the cavities as discussed above. After leaving $\mathcal{C}_1$ the atom crosses a second cavity with a classical microwave field R_2 which is in phase with R_1 and positioned at the distance L from it. The total state change of duration $\delta\tau$ amounts to (cf. [7]) $|\Psi(t_0)\rangle \to |\Psi(t_0 + \delta\tau)\rangle$, where the product state before the atom enters $\mathcal{C}_1$ is given by

$$|\Psi(t_0)\rangle = |g\rangle \otimes |\psi(t_0)\rangle = |g\rangle \otimes \big(c_1(t_0)|1,0\rangle + c_2(t_0)|0,1\rangle\big) \tag{8}$$

and the final entangled state equals

$$|\Psi(t_0+\delta\tau)\rangle = |e\rangle \otimes \left(u_1^e c_1(t_0)|1,0\rangle + u_2^e c_2(t_0)|0,1\rangle\right) + |g\rangle \otimes \left(u_1^g c_1(t_0)|1,0\rangle + u_2^g c_2(t_0)|0,1\rangle\right) . \tag{9}$$

The coefficients in (9) are given by

$$\begin{aligned}
u_1^e &= \frac{1}{2}\sin\left(\frac{\pi}{2}\frac{v_0}{v}\right)\left[e^{i(\varphi_0-\varepsilon)\frac{v_0}{v}}+1\right] \qquad (10)\\
u_2^e &= \frac{1}{2}\sin\left(\frac{\pi}{2}\frac{v_0}{v}\right)\left[e^{i\varphi_0\frac{v_0}{v}}+1\right]\\
u_1^g &= \cos^2\left(\frac{\pi}{4}\frac{v_0}{v}\right) - \sin^2\left(\frac{\pi}{4}\frac{v_0}{v}\right)e^{i(\varphi_0-\varepsilon)\frac{v_0}{v}}\\
u_2^g &= \cos^2\left(\frac{\pi}{4}\frac{v_0}{v}\right) - \sin^2\left(\frac{\pi}{4}\frac{v_0}{v}\right)e^{i\varphi_0\frac{v_0}{v}}
\end{aligned}$$

with $\varepsilon = \frac{\Omega^2}{\delta}\frac{L_{\mathcal{C}}}{v_0}$. v_0 characterizes the Ramsey fields and depends on the length l_r of each of the corresponding cavities and the effective Rabi-frequency Ω_r inside these cavities: $v_0 := 2 l_r\, \Omega_r\, /\pi$. $\varphi_0 := (\omega_r - \omega_{eg})\frac{L}{v_0}$ is the phase shift which is induced by the Ramsey cavities in the case $v = v_0$. An analogous result was obtained in eqn. (A7) of [7] for the initial atomic state being $|e\rangle$. Eqn. (9) shows that the meter states $|e\rangle$ and $|g\rangle$ couple in general to both cavity states $|1,0\rangle$ and $|0,1\rangle$. This is a characteristic trait of a weak measurement.

After the atom has left the second Ramsey field R_2 its energy is finally detected in a projection measurement by field ionization counters D_e and D_g. The state of the composite system after a measurement with result $l \in \{e,g\}$ reads $|\Psi_l(t_0+\delta\tau)\rangle = |l\rangle \otimes |\psi_l(t_0+\delta\tau)\rangle$ with photon state

$$|\psi_l(t_0+\delta\tau)\rangle = |u_1^l|\, c_1(t_0)|1,0\rangle + |u_2^l|\, e^{i(\chi_2^l-\chi_1^l)} c_2(t_0)|0,1\rangle . \tag{11}$$

and $u_j^l = |u_j^l| e^{i\chi_j^l}$ for $j \in \{1,2\}$. Here a global phase factor has been omitted. The probability to obtain the related measurement result l is given by the expectation value of the corresponding projector: $\mathrm{prob}(l) = \langle\, (\,|l\rangle\langle l| \otimes \mathbb{1})\,\rangle_{\Psi(t_0+\delta\tau)}$. Eqn. (11) shows that after the measurement the photon is in general not localized in one of the cavities. The disturbance of the photon state due to the measurement may be small. Because of the Rabi-evolution between the measurements this set up represents no QND measurement of the photon number as it has been in the Brune-Haroche experiment.

Referring to the photon field only, the change of its state due to a single measurement with result l can be expressed by an operation M_l: $|\psi(t_0)\rangle \to |\psi_l(t_0+\delta\tau)\rangle = M_l|\psi(t_0)\rangle$. Like all bounded operators, M_l can be written as "phase" times "modulus" (polar decomposition)

$$M_l = U_l |M_l| \tag{12}$$

with unitary transformation

$$U_l := |1,0\rangle\langle 1,0| + e^{i(\chi_2^l-\chi_1^l)}|0,1\rangle\langle 0,1| \tag{13}$$

and positive operator

$$|M_l| := |u_1^l|\,|1,0\rangle\langle 1,0| + |u_2^l|\,|0,1\rangle\langle 0,1| . \tag{14}$$

The probability to obtain the outcome l is then:

$$\mathrm{prob}(l) = \langle\, M_l^\dagger M_l\,\rangle_{\psi(t_0)} = \langle\, |M_l|^2\,\rangle_{\psi(t_0)} . \tag{15}$$

The $E_l := |M_l|^2$ is also called effect. In this way we obtain e.g. for the probability to measure the energy e: $p_e = p_1|c_1|^2 + p_2|c_2|^2$, where $p_j := |u_j^e|^2$ is fixed by (10).

The effects have the property $E_e + E_g = \mathbb{1}$ and generate a positive operator valued measure (POVM). In the special case where $u_1^e = u_2^g = 1$ and $u_2^e = u_1^g = 0$, the operation $M_l = E_l$ is a projector. If on the other hand $E_l = \mathbb{1}$, no measurement at all has taken place but only an unitary development ($M_l = U_l$). These two cases are the extremes of a sharp and a totally unsharp measurement. By varying the parameters $v\,, v_0\,, \varphi_0$ and ε of the setup all degrees of "weakness" between these two extreme cases as well as the extremes themselves can be reached.

Eqn. (15) shows that the information obtained by the generalized measurement is solely contained in $|M_l|$. This part of the operation M_l in (12) represents at the same time the unavoidable minimal disturbance of the system by the measurement. But our set up causes in addition by means of U_l a purely unitary or Hamiltonian evolution of the state, which modifies the photon state without being necessary for the extraction of information. Since we want to disturb the original state motion as little as possible, we have to install a Hamiltonian feedback mechanism which compensates U_l given by (13). Such a procedure has already been proposed by Wiseman [11]. In our case feedback can be implemented by modifying the set up as follows:

After a measurement beginning at an arbitrary time $t = t_0$ with outcome l an atom prepared in state $|e\rangle$ is sent through the second cavity $\mathcal{C}_2$. As in the case where an atom crosses cavity $\mathcal{C}_1$ the unitary evolution is again governed by the dynamical Stark effect, with the only difference that now the energy shift depends on the number of photons in $\mathcal{C}_2$ instead of $\mathcal{C}_1$:

$$|e\rangle \otimes |\psi_l(t_0+\delta\tau)\rangle \to |e\rangle \otimes U_{\mathcal{C}_2}|\psi_l(t_0+\delta\tau)\rangle , \tag{16}$$

with

$$U_{\mathcal{C}_2} := |1,0\rangle\langle 1,0| + e^{-i\varepsilon_2}|0,1\rangle\langle 0,1| . \tag{17}$$

The combined influence of the measurement and feedback leads to

$$U_{\mathcal{C}_2}|\psi_l(t_0+\delta\tau)\rangle = U_{\mathcal{C}_2} U_l |M_l|\,|\psi(t_0)\rangle . \tag{18}$$

The condition for compensation of U_l in (13) is therefore $U_{\mathcal{C}_2} U_l = \mathbb{1} \Leftrightarrow \varepsilon_2 = \chi_2^l - \chi_1^l$, where χ_j^l may be obtained from (10). This condition demands that the compensating phase $\varepsilon_2 = \frac{\Omega^2}{\delta_f}\frac{L_{\mathcal{C}}}{v_f}$ (f denotes the feedback) has to be

chosen depending on the measurement outcome l. We see two ways to vary ε_2. One is to select an appropriate velocity v_f of the feedback atom sent through the upper cavity $\mathcal{C}_2$. The other possibility consists in setting up a suitable detuning δ_f. This can be done by shifting the atomic energies by means of an static electric field in the cavity $\mathcal{C}_2$ cp. [12]. Please note that it makes no difference whether the atom sent through cavity $\mathcal{C}_2$ is thereafter measured or not because the composite system after the interaction is in a product state.

In order to reach our final aim of monitoring the original Rabi-oscillations of $|\tilde{c}_2(t)|^2$, a sequence of measurements at times $t_n = n\tau$ with $n = 1, 2, 3\ldots$ has to be carried out. Between two consecutive measurements the system evolves undisturbed according to the Hamiltonian (2). The resulting total evolution of the system is given by $c_2(t)$ instead of $\tilde{c}_2(t)$. To process the data obtained in the single measurements we first of all divide the sequence of results with values e and g into groups of N. From each so called "N-series" we extract the relative frequency $r := N_e/N$ of the number N_e of e-results. It turns out [2] that its expectation value $\mathcal{E}(r)$ is related to the value which $|c_2|^2$ assumes immediately before the start of the N-series by

$$|c_2|^2 = \frac{\mathcal{E}(r) - p_1}{\Delta p} \tag{19}$$

with $\Delta p := p_2 - p_1 = |u_2^e|^2 - |u_1^e|^2$ of (10). In a sequence of measurements on a single radiation field we do not have access to the expectation value $\mathcal{E}(r)$, which refers to an ensemble. Instead we insert $r(t_0)$ into the right hand side of equation (19) and obtain thus a "best guess" of $|c_2(t_0)|^2$ at time t_0 when the first measurement of the respective N-series began:

$$\mathrm{G}_2(t_0) = \frac{r(t_0) - p_1}{\Delta p}\,. \tag{20}$$

The possible values of $\mathrm{G}_2(t_0)$ are distributed around $|c_2(t_0)|^2$ and may be negative. This estimation of $|c_2(t_0)|^2$ can be good only if the duration of the N-series $N\tau$ is much smaller than the period T_R of the oscillations of the system. The sequence of G_2 at various times serves as the final readout of the sequential measurement.

We have two competing influences on the system: The strength of the original dynamics is proportional to g or $\Omega_R = 2\pi/T_R$. The measurements on the other hand hinder this dynamics the more the stronger they are and the quicker they are repeated with frequency $1/\tau$. The Zeno effect demonstrates this! A measure for the disturbance due to the sequence of measurements is the decoherence time T_D, the time after which the off-diagonal elements of the density matrix (in the $|\varphi_1\rangle$, $|\varphi_2\rangle$ basis) have decayed to $1/e$ of their original value. In a subsequent paper we will show that $T_D = 8\frac{p_0(1-p_0)}{(\Delta p)^2}\tau$ with $p_0 := (p_1+p_2)/2$. In order to have a high resolution in the sequential measurement it would be desirable to have a small decoherence time (T_D is proportional to the time it takes to distinguish between the states $|\varphi_1\rangle$ and $|\varphi_2\rangle$, cp. [2]). On the other hand the influence of the measurement should not dominate the evolution. We therefore require the decoherence time to be maximally as great as the Rabi time. A numerical analysis [2] showed that for our purpose a favorable balance of information gain and disturbance is obtained if the so called fuzziness $f := \frac{\pi T_D}{2T_R}$ is adjusted to be close to one: $f \approx 1$. In fact it suffices to choose f in the interval $0.75 \lesssim f \lesssim 1.5$. The experimental parameters ε, φ_0, v_0, v and τ have to be fixed correspondingly.

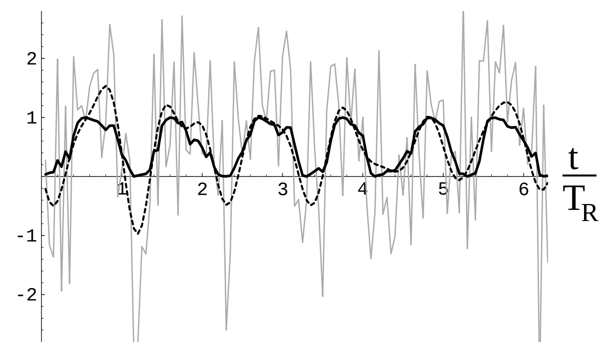

FIG. 3. Under a sequence of appropriate weak measurements the measurement readout G_2 (grey curve) is correlated with the state evolution $|c_2(t)|^2$ (black curve). This becomes evident after noise reduction (dashed curve) of the readout G_2, which was carried out taking into account approximately 12 Rabi-cycles. Parameter values: $\frac{v}{v_0} \in [1.25 - 0.125, 1.25 + 0.125]$, $\varepsilon = 0.068 \cdot \pi$, $\varphi_0 = \pi$, $\tau = 0.002T_R$ and $N = 25$, which lead to an average fuzziness of $f = 0.98$.

We have simulated numerically all the processes described above including the feedback. In a realistic experiment the velocity v of the probing atoms and the feedback atoms will vary from one single measurement to the other. We have accordingly tolerated the velocities v to fluctuate uniformly by $\pm 10\%$ about the desired mean value. The resulting dynamics of the state under the influence of stroboscopically applied weak measurements is given by the $|c_2(t)|^2$-curve (black) in Fig. 3. The measurement readout which is defined as best guess $\mathrm{G}_2(t)$ of $|c_2(t)|^2$ (grey curve) has been further processed to the noise reduced G_2-curve (dashed). The noise reduction procedure consists essentially in a time-averaging of the readout. Details are described in Appendix C of [2].

We find a high correlation including the phase between the noise reduced G_2-curve and the $|c_2|^2$-curve. The actual evolution of the state is therefore well monitored in time. The $|c_2|^2$-curve reflects the fact that the original Rabi-oscillations have been disturbed by the measurement, though they are only slightly modified.

Fig. 4 shows the power spectrum of the $|c_2|^2$-curve (black) and the measurement readout G_2 (grey). Both curves are peaked at the Rabi frequency Ω_R.

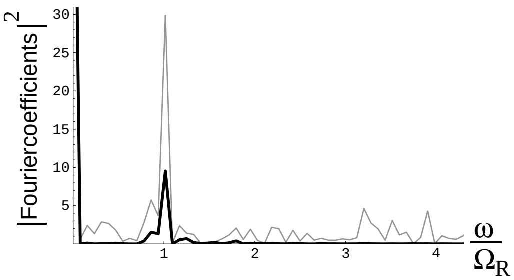

FIG. 4. Power spectra of $|c_2(t)|^2$ (black) and $\mathrm{G}_2(t)$ (grey).

All parameters used in Fig. 3 and 4 are realistic (cf. [7]) apart from the number of Rabi cycles and the efficiencies of the detector and the feedback mechanism. In what follows, we are going to address these remaining problems.

The number of Rabi cycles which can be monitored depends crucially on the mean lifetime t_{cav} of the coupled cavities which has so far been assumed to be infinite. At the required frequency of 50 GHz a $t_{\text{cav}} = 0.1$ s seems to be feasible (cf. [7]). The time of flight of an atom with velocity $v = 1000$ m/s through cavity $\mathcal{C}_1$ amounts to $L_C/v = 10\mu$s. We therefore assume that a time $\tau = 100\mu$s between two consecutive measurements suffices to send a feedback atom through cavity $\mathcal{C}_2$ between the measurements. Then 1000 measurements can be made within the mean lifetime t_{cav} of the cavities. For $T_R/\tau = 500$ as assumed in Fig. 3, this corresponds to monitoring the first two Rabi cycles. As can be seen in Fig. 3 the readout of the measurement G_2 (grey curve) oscillates about the evolution of the component $|c_2|^2$ (black curve), so that also for two Rabi cycles the evolution of $|c_2|^2$ can be approximately recovered by averaging G_2 over an appropriate timescale.

A more serious problem arises when some of the feedback atoms are not sent through cavity $\mathcal{C}_2$ and thus the unitary part of the back-action U_l is not always compensated. In our simulations we obtained still good results with 4 percent of the feedback atoms missing while for more than 7 percent the evolution of $|c_2|^2$ is significantly disturbed. Since the preparation of the Rydberg atoms in the Haroche experiment is a random process, feedback atoms might be missing. This problem could be solved by sending a high number of atoms at once through cavity $\mathcal{C}_2$, each with large detuning, i.e., causing a small phase shift of the radiation field. The mean number $\bar{n}$ of atoms sent in one burst should lead to a total phase shift, which compensates U_l. Since the standard deviation divided by the mean number $\Delta n/\bar{n}$ can be made arbitrarily small by increasing $\bar{n}$, U_l can be compensated very precisely.

Another crucial point is the detector efficiency. The measurement results are not so sensitive to the loss of information. It is the disturbance of the Rabi oscillations as a consequence of not knowing how to prepare the feedback atoms which mainly hinders the visualization of the photon dynamics. The problem can be solved by employing a detector with more than 96% efficiency (see above). If only detectors with moderate efficiencies are available there is another solution. It consists in choosing the experimental parameters in (10) such that the measurement statistics become Poissonian:

$$u_1^e = 0,\ u_2^e = \tilde{\epsilon},\ u_1^g = 1,\ \text{and}\ u_2^g = \sqrt{1-\tilde{\epsilon}^2}, \tag{21}$$

where $\tilde{\epsilon} = \sin(\pi v_0/2v)$ should be small. In this case we do not need feedback because $U_l = \mathbb{1}$ for both $l = e$ and $l = g$. Most measurement results will be "g ", in which case the change of the state will be of higher order in the small parameter $\tilde{\epsilon}$. The result "e " is likely only when $|c_2|^2 \approx 1$ and then the state change is also very small. The disadvantage of the Poissonian method is that the quality of the monitoring suffers from the Poissonian statistics. The measurement results essentially indicate only the parts of the state evolution where $|c_2|^2 \approx 1$. For detector efficiencies higher than 96% the method with feedback atoms shows clearly better results. For efficiencies below 96% it is preferable to avoid the necessity of feedback atoms. Fig. 5 displays a simulation of a sequence of measurements with Poissonian statistics for a detector efficiency of 60%. Roughly the same percentage of the maxima of the Rabi oscillations are indicated by the readout G_2. As in the case with feedback we allowed the velocities of the Rydberg atoms to fluctuate uniformly by $\pm 10\%$ around the mean value $\bar{v}$.

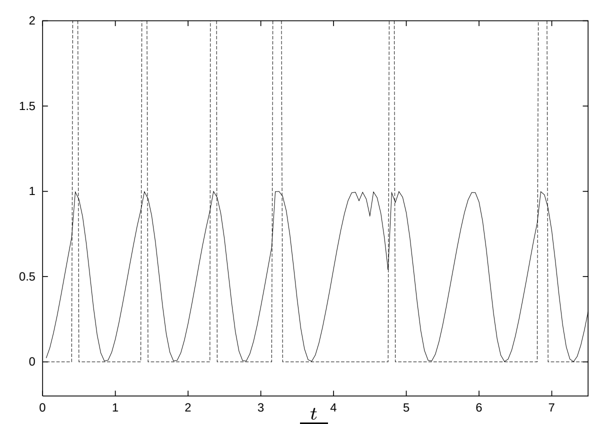

FIG. 5. Simulation of a sequence of weak measurements with Poissonian statistics and a detector efficiency of 60%. The back-action of these measurements has no unitary part. The dashed and the black curve represent G_2 and $|c_2(t)|^2$ respectively. Parameter values: $\frac{v}{v_0} \in [20-2, 20+2]$, $\varepsilon = \pi\frac{\bar{v}}{v_0}$, $\varphi_0 = 0$, $\tau = 0.002T_R$ and $N = 25$, which lead to an average fuzziness of $f = 2.04$.

To sum up: Using the feedback mechanism for detector efficiencies higher than 96% the original Rabi-oscillations are well visualized in phase and frequency. For moderate detector efficiencies an alternative method leads to a moderate visibility of the Rabi-oscillations.

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